The complete relativistic kinetic model of violation of symmetry in isotopic expanding plasma and production of baryons in hot Universe. II. Numerical model: X-boson distribution function.

Yu.G.Ignat'ev, K.Alsmadi
Kazan State Pedagogical University,
Mezhlauk str., 1, Kazan 420021, Russia

Abstract

In terms of proposed by authors general-relativistic kinetic model of baryon production in expanding primordially symmetrical hot Universe calculates distribution function of extra-massive bosons and concerned with it variables.

1 Introduction

In previous paper [1] in terms of theory , developed earlier by one of the authors,[1] was proposed an accurate kinetic model of baryon production in primordially baryon-symmetrical Universe on basis of field model of interaction of particles of type SU(5). Here and further references to formulas of authors previous paper [1] are indicate as (I,N), where N - number of formula of stated paper.

2 Weak violence of charge symmetry in standard SU(5) model

Using weakness of violence of CP-symmetry

$$\Delta r = r - \bar{r} \lesssim 10^{-6}$$

and thereby smallness of chemical potentials ($\lambda_a \lesssim \Delta r \ll 1$), we can reduce referred above system of integro-differential equations to a system of linear integro-differential equations. In present paper we will consider simple model of baryogenesis, in which CP-invariance is violated in decay channels of only one type of bosons, X-boson. In this case equations (I.40), (I.42) assume following form:

$$\begin{aligned}\dot{f}_X + f_X \frac{a(t)S^2}{2\pi\sqrt{a^2(t)m_X^2+\mathbb{P}^2}} \times \\ \times\left[\bar{r}\beta(d) + (1-\bar{r})\beta(-u)\right] = \\ \frac{a(t)S^2}{2\pi\sqrt{a^2(t)m_X^2+\mathbb{P}^2}} \times \\ \left[\bar{r}\beta(d)f_X^0(2d) + (1-\bar{r})\beta(-u)f_X^0(-2u)\right];\end{aligned} \qquad (1)$$

[1] see Yu.G.Ignat'ev, [2], [3]

$$\Delta\dot{\mathcal{N}}_\alpha = \frac{2a(t)S^2N_X}{3(2\pi)^3}\int\limits_0^\infty \frac{\mathbb{P}^2 d\mathbb{P}}{\sqrt{a^2(t)m_X^2+\mathbb{P}^2}}\times$$
$$\big\{(1-\bar{r})\beta(u)[f_{\bar{X}}-f_X^0(2u)]-$$
$$-(1-r)\beta(-u)[f_X-f_X^0(-2u)]\big\}\,; \quad (2)$$

$$\Delta\dot{\mathcal{N}}_\kappa = \frac{2a(t)S^2N_X}{3(2\pi)^3}\int\limits_0^\infty \frac{\mathbb{P}^2 d\mathbb{P}}{\sqrt{a^2(t)m_X^2+\mathbb{P}^2}}\times$$
$$\big\{r\beta(d)[f_{\bar{X}}-f_X^0(2d)]-\bar{r}\beta(-d)[f_{\bar{X}}-f_X^0(-2d)]\big\}\,. \quad (3)$$

Here separately wrote out equations for anoquarks- (2), and katoquarks -(3). At derivation of these equation was allowed that:

$$\beta(x,x)=2\beta(x). \quad (4)$$

Let us write out formulas for coefficients that belong to these equations in linear in $\Delta r, \lambda$ approximation. In zero approximation:

$$\beta(0)=\beta_0(\mathbb{P},t), \quad (5)$$

where:

$$\beta_0(\mathbb{P},t)=\frac{T}{p}\ln\frac{1+\exp(p_+/T)}{\exp(p/2T)[1+\exp(p_-/T)]} \quad (6)$$

and it is necessary to substitute:

$$\frac{p_\pm}{T}=\frac{1}{2}\frac{\sqrt{a^2(t)m_X^2+\mathbb{P}^2}\pm\mathbb{P}}{\mathcal{T}}. \quad (7)$$

Further:

$$f_X^0(\xi)=f_X^0(0;\mathbb{P},t)+\xi\ \frac{\exp(E_X/T)}{[\exp(E_X/T)-1]^2}, \quad (8)$$

where:

$$f_X^0(0;\mathbb{P},t)=[\exp(E_X/T)-1]^{-1} \quad (9)$$

and

$$\frac{E_X}{T}=\frac{\sqrt{a^2(t)m_X^2+\mathbb{P}^2}}{\mathcal{T}}.$$

Integrating kinetic equations for X-bosons (1) with initial conditions (I.43) in linear in $\Delta r, \lambda$ approximation we will determine:

$$f_X(\mathbb{P},t)=f_X^0(0;\mathbb{P},0)+\delta f_0(\mathbb{P},t)+\delta f(\lambda), \quad (10)$$

where $\delta f(\lambda)$ - linear in λ functional, hence:

$$\delta(-\lambda) = -\delta(\lambda), \tag{11}$$

and it doesn't contribute in final output; and δf - deviation from equilibrium of distribution function:

$$\delta f(\mathbb{P}, t) = -e^{-\Phi(\mathbb{P},t)} \int\limits_0^t e^{\Phi(\mathbb{P},t')} \dot{f}_0(0; \mathbb{P}, t') dt' \tag{12}$$

- deviation from equilibrium of boson distribution function in symmetrical plasma $\lambda = 0$, and incorporated notation:

$$\Phi(\mathbb{P}, t) = \frac{1}{\tau_0} \int\limits_0^t \frac{a(t')\beta_0(\mathbb{P}, t')dt'}{\sqrt{a^2(t') + \mathbb{P}^2/m_X^2}}; \tag{13}$$

τ_0 - proper time of free X-boson's decay:

$$\tau_0 = \frac{4\pi m_X}{s^2} \sim \frac{3}{2}(m_X \alpha)^{-1}. \tag{14}$$

Since fermions lay in condition of and are ultrarelativistic, their concentrations are equal:

$$\mathcal{N} = \frac{1}{\pi^2} \int\limits_0^\infty \frac{1}{\exp(-\lambda + \mathbb{P}/\mathcal{T}) - 1} \mathbb{P}^2 d\mathbb{P}. \tag{15}$$

In conditions of weak violence of CP - invariance, when $\lambda_a \ll 1$ we will receive from here approximately, separating at degrees of smallness λ :

$$\mathcal{N} \approx \frac{3}{2}\frac{\mathcal{T}^3\zeta(3)}{\pi^2} + \lambda\frac{\mathcal{T}^3}{6} \Rightarrow$$

$$\Delta N_a \approx \lambda_a \frac{\mathcal{T}^3}{3}. \tag{16}$$

In standard SU(5) model probabilities of X-boson's decay in lepton, (qe), and quark, $(\bar{q}\bar{q})$, channels are the same [2], i.e.:

$$1 - 2r = O^1(\Delta r). \tag{17}$$

Subject to this factor and relation (16) *in standard* SU(5) model from equations for anoquarks- (2), and katoquarks -(3) we can receive one closed first-order equation on variable $B = u + 2d$, i.e., on excessive concentration of baryons:

$$\Delta\mathcal{N}_B = \frac{1}{3}\mathcal{N}_B \mathcal{T}^3 :$$

[2] subject to colors and charms

$$\frac{d}{dt}\Delta\mathcal{N}_B + \Delta\mathcal{N}_B\frac{2N_X}{\pi^2\mathcal{T}^3}\int\limits_0^\infty \mathbb{P}^2\dot{\Phi}f_0\beta_0 d\mathbb{P} =$$

$$= \frac{2\Delta r N_X}{3\pi^2}\int\limits_0^\infty \mathbb{P}^2\dot{\Phi}\beta_0\delta f_0 d\mathbb{P}, \qquad (18)$$

integrating which subject to initial conditions (I.43), we will obtain:

$$\Delta\mathcal{N}_B(\infty) = \frac{4\Delta r N_X}{3\pi^2}\int\limits_0^\infty \exp\left(-\int\limits_t^\infty \Psi(t')dt'\right)G(t)dt, \qquad (19)$$

where

$$\Psi(t) = \frac{2N_X}{\pi^2\mathcal{T}^3}\int\limits_0^\infty \mathbb{P}^2\dot{\Phi}f_0\beta_0 d\mathbb{P}, \qquad (20)$$

$$G(t) = \frac{1}{\pi^2}\int\limits_0^\infty \mathbb{P}^2\dot{\Phi}\delta f d\mathbb{P}. \qquad (21)$$

Now it remains to calculate variable $\delta_S = \delta n_B/S$, where entropy density of ultrarelativistic gas is equal:

$$S = \frac{2\pi^2}{45}NT^3 \Rightarrow \mathcal{S} = \frac{2\pi^2}{45}N\mathcal{T}^3. \qquad (22)$$

By that the task is formally solved. We should note that in contrast to papers [4]-[6], in which distribution function of X-bosons was modelled by quasi-hydrodynamic distribution and was obtained by method of numerical integration of kinetic equations, distribution function of X-bosons in present article obtains by strict integration of kinetic equations and $B(\infty)$ determines in quadratures:

$$\delta_S = \frac{\Delta\mathcal{N}_B}{\mathcal{S}}. \qquad (23)$$

3 Analysis of solution

Further we will assume that firstly X-bosons decay in general on intermediate stages of expansion, when $T \sim m_X$, and, secondly that part of X-bosons sufficiently small in comparison with general number of particles: $N_X \ll N$, so with enough accuracy grade we can assume cosmological plasma on decay stage of X-bosons is ultrarelativistic. This gives us laws of variation of dimensioned factor and temperature in due course:

$$a(t) = a_0\sqrt{t}; \quad T = \mathcal{T}_0\frac{1}{\sqrt{t}}, \qquad (24)$$

where $\mathcal{T}_0$ (. (I.44)) is plasma's temperature in Planck units on Planck moment of time:

$$\mathcal{T}_0 = \left(\frac{45}{16\pi^3 N}\right)^{1/4}. \qquad (25)$$

Let us explore obtained in previous section linear solution for that we will come over from variables t, p to new variables η, ξ:

$$t = \tau_0 \eta; \qquad (26)$$

$$p = \frac{1}{\sqrt{\eta}} m_X \xi; \quad \Rightarrow \mathbb{P} = m_X \xi \sqrt{\tau_0}. \qquad (27)$$

where τ_0 - proper time of free X-boson's decay (14), in that way to valuation of dimensionless time $\eta = 1$ it corresponds time $t = \tau_0$. Let us incorporate dimensionless parameter σ [3], depending from constants of field theory [3]:

$$\sigma = \frac{m_X}{T(\tau_0)} = \frac{m_X\sqrt{\tau_0}}{\mathcal{T}_0} = \frac{\chi\sqrt{m_X}}{\sqrt{\alpha}\mathcal{T}_0}, \qquad (28)$$

which is equal to relation of X-boson's mass to temperature at a point of it's half-decay, where χ - dimensionless parameter of 1 order, depending from parameters of field theory, α - coupling constant (0.1 0.01).

Then we will receive expressions:

$$\frac{p}{T} = \sigma\xi; \quad \frac{E}{T} = \sigma\sqrt{\eta + \xi^2}; \quad \frac{p_\pm}{T} = \sigma(\sqrt{\eta + \xi^2} \pm \xi) \qquad (29)$$

$$f_0(t, \mathbb{P}) = \left[1 + \exp(\sigma\sqrt{\eta + \xi^2})\right]^{-1} \qquad (30)$$

$$\beta_0(\mathbb{P}, t) = \beta_0(\xi, \eta, \sigma) =$$
$$= \frac{1}{\xi\sigma} \ln \frac{1 + \exp(\sigma/2(\sqrt{\eta + \xi^2} + \xi))}{\left[1 + \exp(\sigma/2(\sqrt{\eta + \xi^2} - \xi))\right] \exp(\sigma\xi/2)} \qquad (31)$$

On fig. 1 is shown relation of statistical factor $\beta_0(\xi, \eta, \sigma)$ (31) from parameters σ $\xi = p$.

[3] Incorporated in [3] parameter σ is equal to quadrate of σ, which uses in present article.

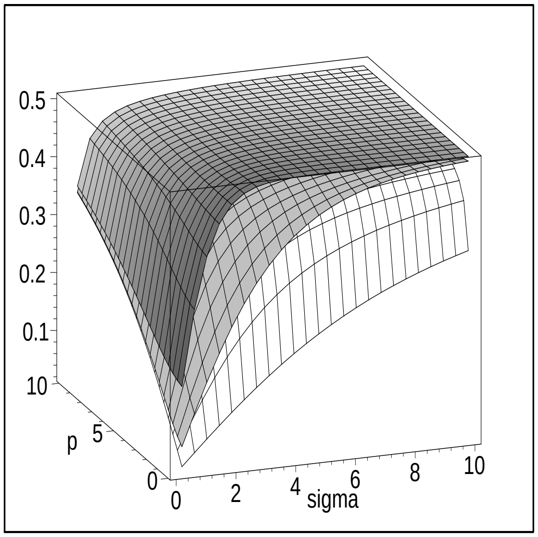

Fig.**1.** *Function $\beta_0(\xi, \eta, \sigma)$ for values of time $\eta = 0, 1, 10$ (bottom-up) .*

3.1 Boltzmann distribution

Let us calculate function $\Phi(\mathbb{P}, t)$ (13), going over variables ξ, η according to formulas (26), (27):

$$\Phi(\mathbb{P}, t) = \Phi(\xi, \eta) = \int\limits_0^\eta \frac{\beta_0(\xi, \eta, \sigma))\sqrt{\eta'}d\eta'}{\sqrt{\eta + \xi^2}}. \tag{32}$$

In Boltzmann approximation, substituting statistical factor $\beta_0 \;\; \frac{1}{2}$ and executing integration we will receive:

$$\Phi(\xi, \eta) = \frac{1}{8}(\sqrt{\eta} + \sqrt{\eta + \xi^2})^2 -$$

$$- \frac{1}{8}\frac{\xi^4}{(\sqrt{\eta} + \sqrt{\eta + \xi^2})^2} - \xi^2 \ln \frac{(\sqrt{\eta} + \sqrt{\eta + \xi^2})^2}{\xi}. \tag{33}$$

3.2 Deviation of X-bosons from equilibrium

Substituting function $\Phi(\xi, \eta)$ in form (33)in expression (12) for X-boson's distribution function deviation from equilibrium and going over variables (26), (27), we will obtain:

$$\delta f(\xi, \eta, \sigma) = \frac{1}{2}\sigma \exp(-\Phi(\xi, \eta)) \int\limits_0^\eta \frac{\exp\left(\Phi(\xi, \eta') + \sigma\sqrt{\eta' + \xi^2}\right) d\eta'}{[1 + \exp(\sigma\sqrt{\eta' + \xi^2})]^2\sqrt{\eta' + \xi^2}}. \tag{34}$$

On Fig. 2, 3 results of numerical integration of expression (34) for relative deviation of distribution function from equilibrium, $\delta f/f0$, subject to time, η, and momentum, ξ for value $\sigma = 1$ are shown. Let us notice that fermi-distribution is substituted to boltzmann distribution only in expression (32) for function $\Phi(\xi, \eta)$).

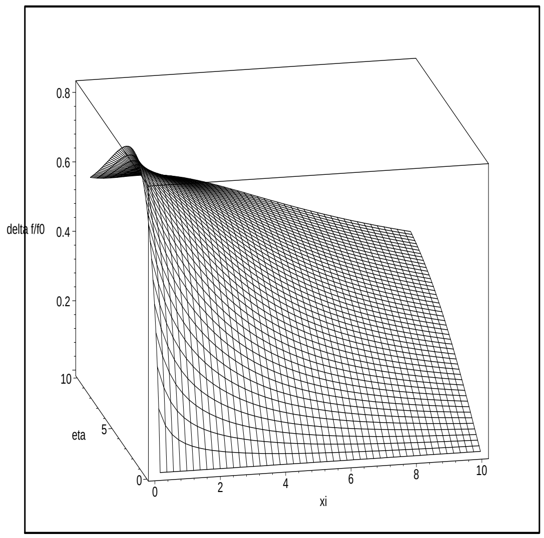

Fig.**2.** *Relative deviation of distribution function of X-bosons from equilibrium* $\delta f(\xi, \eta, \sigma)/f_0$ *under* $\sigma = 1$.

From dduced results follows that:

1. Maximum of relative deviation of distribution function from equilibrium , $(\delta f/f_0)$, falls at lesser values of momentum coordinate $\xi \leq 1$;

2. On initial stages relative deviation of distribution function from equilibrium grows with time η;

3. However at achievement certain critical sufficiently great value of time relative deviation of distribution function from equilibrium begins to decrease, - it associates with a fact, that by this moment of time X-bosons start to decay;

4. At very great times weak maximum of relative deviation of distribution function from equilibrium appears and starts to migrate to the range of great values of momentum coordinate.

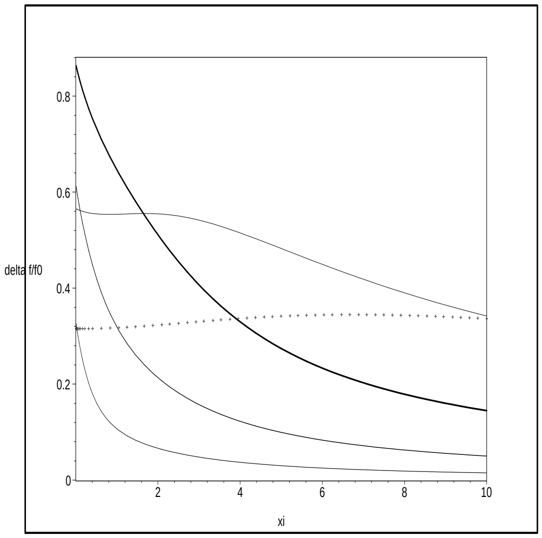

Fig.**3.** *Evolution of relative deviation of distribution function of X-bosons from equilibrium*
$\delta f(\xi,\eta,\sigma)/f_0$ *under* $\sigma = 1$. *Thin line -* $\eta = 0.3$, *normal line -* $\eta = 1$, *heavy line -* $\eta = 3$, *thin dotted line -* $\eta = 10$, *heavy dotted line -* $\eta = 20$.

Let us notice that specific features of behavior of distribution function's deviation were exhibit but were not explored in paper [2]. It is necessary to allow that under great values of time coordinate η, absolute magnitude of density of X-bosons number becomes vanishingly small. On Fig. 4 - 6 are shown results of numeric integration of expression (34) for density of X-bosons deviation from equilibrium, $\xi^2\delta f$, subject to time, η, and momentum, ξ for values $\sigma = 0.3, 1, 3$.

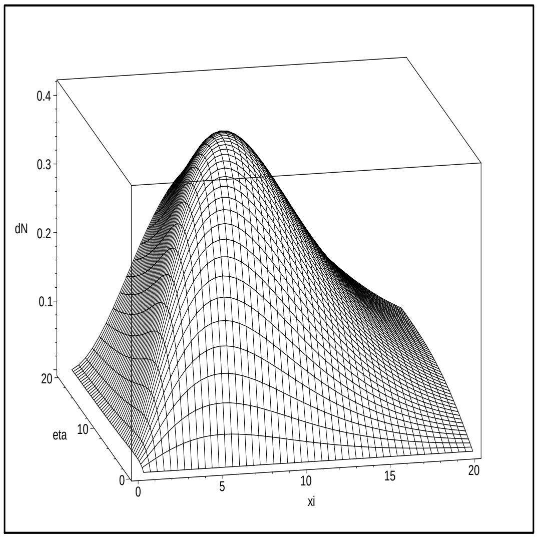

Fig.**4.** *Density of non-equilibrium X-bosons number* $dN = \xi^2 \delta f(\xi, \eta, \sigma)$ *under* $\sigma = 0, 3$.

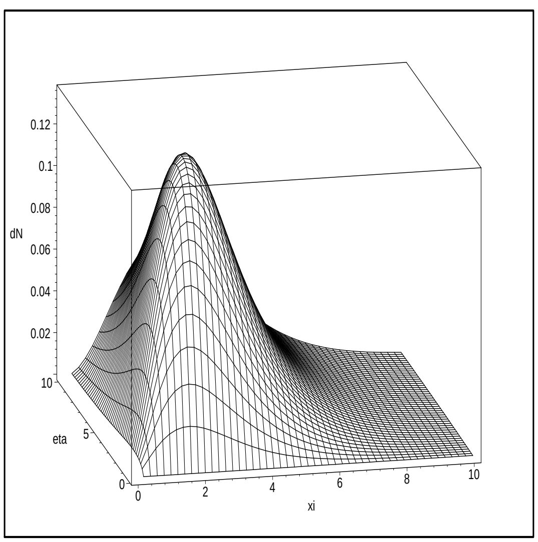

Fig.**5.** *Density of non-equilibrium X-bosons number* $dN = \xi^2 \delta f(\xi, \eta, \sigma)$ *under* $\sigma = 1$.

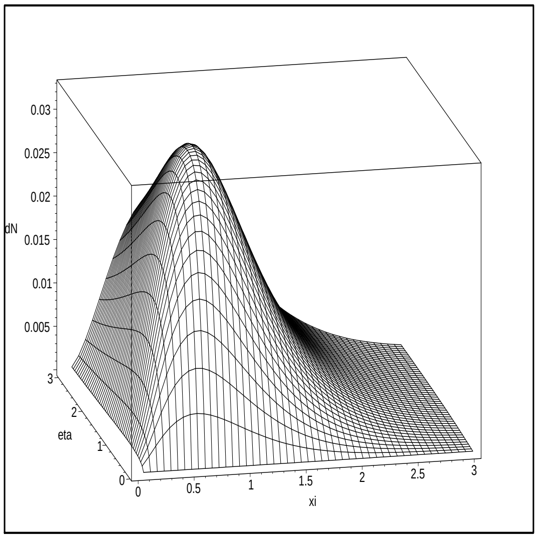

Fig.**6.** *Density of non-equilibrium X-bosons number* $dN = \xi^2 \delta f(\xi, \eta, \sigma)$ *under* $\sigma = 3$.

On Fig. 7- 9 are shown results of numeric integration for evolution of X-bosons density's deviation from equilibrium, $\xi^2 \delta f$, calculated by formula (34), for values $\sigma = 3, 1, 0.3$.

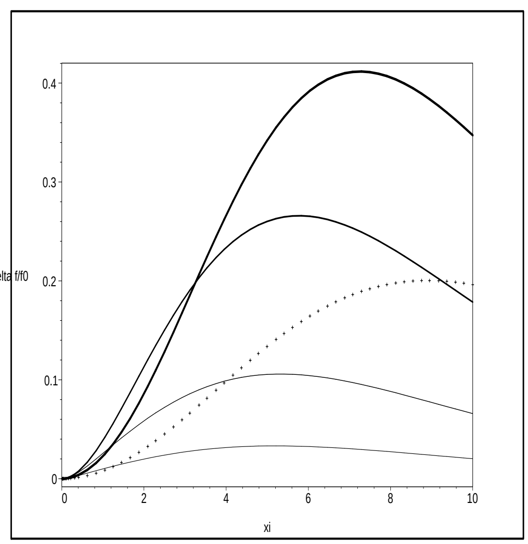

Fig.**7.** *Evolution of number density of non-equilibrium X-bosons* $dN = \xi^2 \delta f(\xi, \eta, \sigma)$

under $\sigma = 0.3$*. Thin line -*$\eta = 0.3$*, medium line -* $\eta = 1$*, heavy line -* $\eta = 3$*, the heaviest line -* $\eta = 10$*, dotted line -* $\eta = 30$*.*

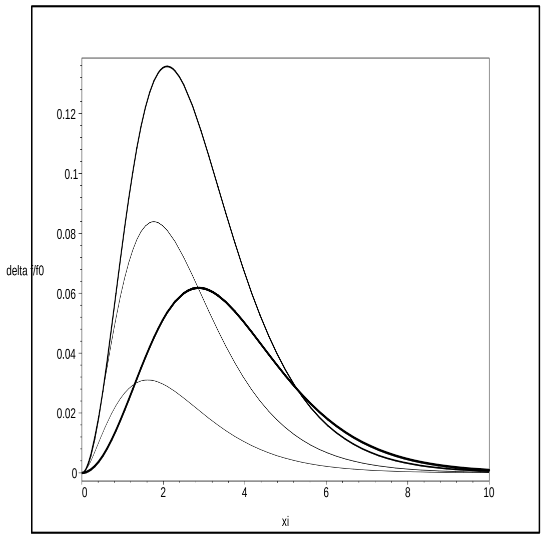

Fig.**8.** *Evolution of number density of non-equilibrium X-bosons* $dN = \xi^2 \delta f(\xi, \eta, \sigma)$ *under* $\sigma = 1$*. Thin line -*$\eta = 0.3$*, medium line - *$\eta = 1$*, heavy line -* $\eta = 3$*, the heaviest line -* $\eta = 10$*.*

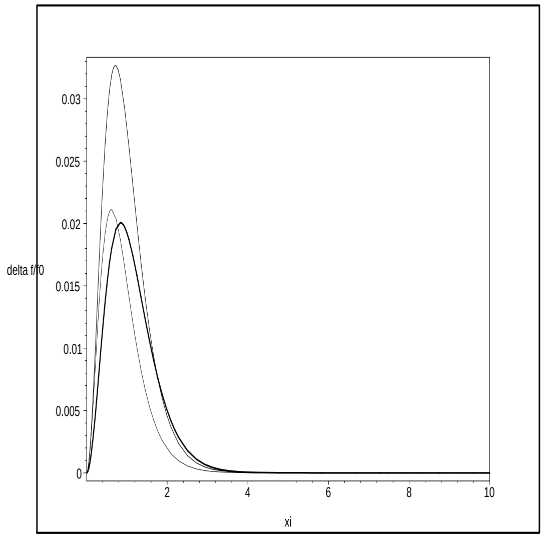

Fig.**9.** *Evolution of number density of non-equilibrium X-bosons* $dN = \xi^2\delta f(\xi,\eta,\sigma)$ *under* $\sigma = 3$. *Thin line -*$\eta = 0.3$, *medium line -* $\eta = 1$, *heavy line -* $\eta = 3$.

3.3 Exponential factor in formula (19)

Further from formulas (19)-(22) it is seen that final solution defines not by function itself $\Phi(\mathbb{P},t)$, but by it's time derivative, calculating which we will find, proceeding to variables ξ,η according to formulas (26), (27):

$$\dot{\Phi}(\mathbb{P},t) = \frac{1}{\tau_0}\frac{\sqrt{\eta}\beta_0(\eta,\sigma,\xi)}{\sqrt{\eta+\xi^2}} \tag{35}$$

or, in Boltzmann approximation:

$$\dot{\Phi}(\mathbb{P},t) = \frac{1}{2\tau_0}\frac{\sqrt{\eta}}{\sqrt{\eta+\xi^2}} \tag{36}$$

Thus, calculating an integral by momentums we will receive expression for function $\Psi(\eta,\sigma)$ (20) in Boltzmann approximation:

$$\Psi(\eta,\sigma) = \frac{N_X\sigma^2\eta}{\pi^2\tau_0}K_1(\sigma\sqrt{\eta}), \tag{37}$$

where $K_n(z)$ - modified Bessel function (Bessel imagunary function) [**?**]:

$$K_\nu(z) = \int\limits_0^\infty e^{-z\cosh u}\cosh\nu u du, \tag{38}$$

at that following recurrence equations are correct:

$$K_{\nu+1}(z) - K_{\nu-1}(z) = \frac{2\nu}{z} K_\nu(z). \tag{39}$$

Thus, exponential factor in formula (19) is equal:

$$\Psi_e(\eta,\sigma) = \exp(-\int\limits_t^\infty \Psi(t',\sigma)dt') = \exp\left(-\frac{2N_X}{\pi^2\sigma^2}\int\limits_{\sigma\sqrt{\eta}}^\infty x^3 K_1(x)dx\right). \tag{40}$$

Considering that:

$$\int\limits_0^\infty x^3 K_1(x)dx = \frac{3\pi}{2}, \tag{41}$$

it is not hard to receive limitary relation:

$$\Psi_e(\eta,\sigma) \approx \exp\left(-\frac{3N_X}{\pi\sigma^2}\right), \qquad (\sigma\sqrt{\eta} \to 0), \tag{42}$$

as well as asymptotic expansion of function $\Psi_e(\eta,\sigma)$ at small values of argument $\sigma\sqrt{\eta}$:

$$\Psi_e(\eta,\sigma) \approx \exp\left(-\frac{3N_X}{\pi\sigma^2} + \frac{2N_X\sigma\eta^{3/2}}{3\pi^2}\right), \quad (\sigma\sqrt{\eta} \to 0). \tag{43}$$

In other limitary case accounting asymptotic expansion of Bessel functions at great values of argument we will receive:

$$\Psi_e(\eta,\sigma) \approx \exp\left(-\frac{3N_X}{\pi^2}\sqrt{2\pi\sigma}\eta^{5/4}\mathrm{e}^{-\sigma\sqrt{\eta}}\right)$$
$$\approx 1 - \frac{3N_X}{\pi^2}\sqrt{2\pi\sigma}\eta^{5/4}\mathrm{e}^{-\sigma\sqrt{\eta}} \qquad (\sigma\sqrt{\eta} \to \infty). \tag{44}$$

On Fig.10 dependence of exponential factor from variables η, σ, received by numerical integration for $N_X = 1$, is shown.

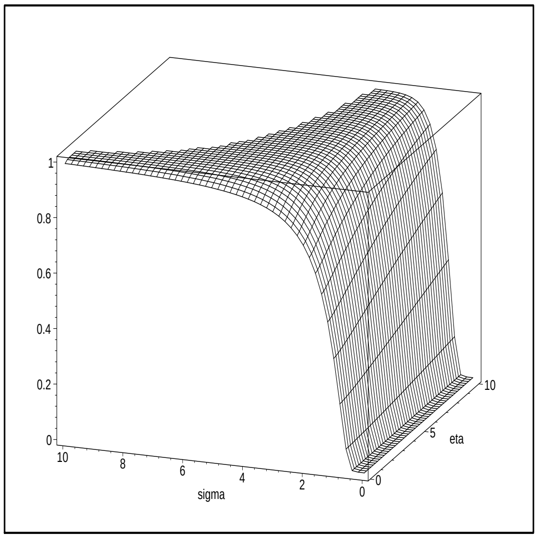

Fig.**10.** *Function* $\Psi_e(\eta,\sigma) = \exp(-\int\limits_t^\infty \Psi dt')$ *at* $N_X = 1$.

On Fig. 11 is shown an evolution of exponential factor $\Psi_1(\eta,\sigma)$ at various values of parameter σ,

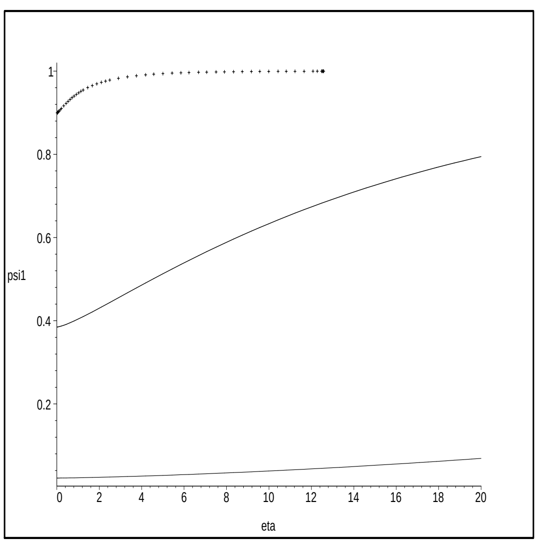

Fig.**11.** *Function* $\Psi_e(\eta,\sigma) = \exp(-\int\limits_t^\infty \Psi dt')$ *at* $N_X = 1$. *Thin line:* $\sigma = 0.3$,

medium line: $\sigma = 1$, *dotted line:* $\sigma = 3$.

On Fig. 13 influence of number of X-boson types on exponential factor, N_X is shown.

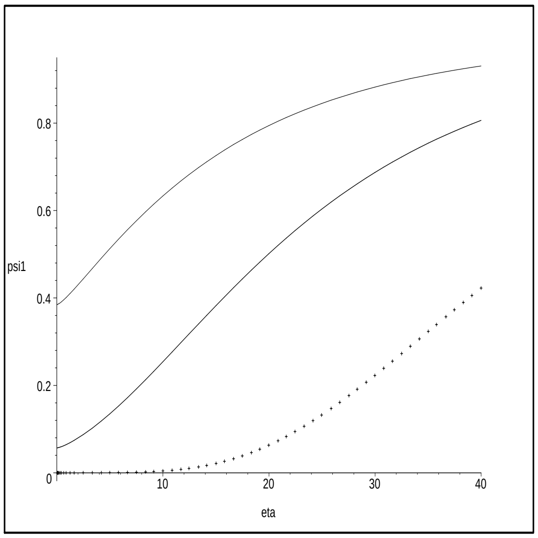

Fig.**12.** *Influence of number of X-boson types,* N_X*, on exponential factor* $\Psi_1(\eta, \sigma) = \exp(-\int\limits_t^\infty \Psi dt')$ *at* $\sigma = 1$. *Thin line:* $N_X = 1$, *heavy line:* $N_X = 3$, *dotted line:* $N_X = 12$.

On Fig. 13 are shown values of exponential factor, calculated by numerical methods and using asymptotic estimates (43), (44).

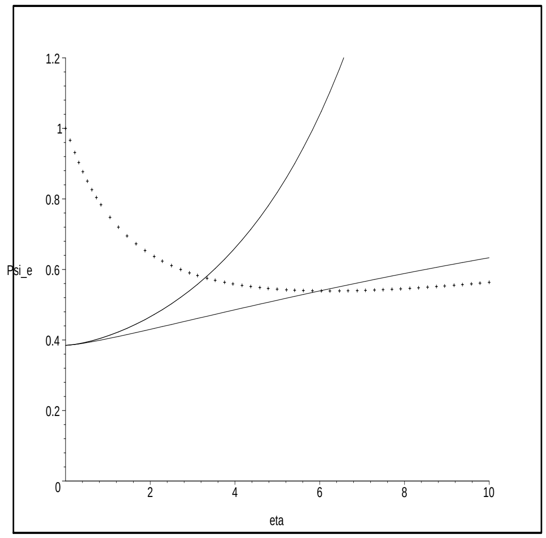

Fig.**13.** *Comparison of exponential factor values, calculated by numerical methods (thin line) and by asymptotic estimates (43) -heavy line and (44) - dotted line.*

References

[1] Yu.G.Ignatyev, K.Alsmadi. *Gravitation & Cosmology*, Vol. 11, No 3, 2005, pp.

[2] Yu.G. Ignat'ev, *Astronom. Journ*, **62**, 633 (1985) (in Russian).

[3] Yu.G. Ignat'ev, *in: "Classical and Quantum-statistical problems of relativistic theory of gravitation"*, Izdatelstvo KSPI, Kazan, 1991 (in Russian).

[4] J.M. Fry, K.A. Olive, M.S. Turner, *Phys. Rev.D* **22**, 2953 (1980).

[5] J.M. Fry, K.A. Olive, M.S. Turner, *Phys. Rev.D* **22**, 2977 (1980).

[6] J.M. Fry, K.A. Olive, M.S. Turner, *Phys. Rev. Lett* **45**, 2074 (1980).